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# Quadratic phase of tightly focused wavefronts

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**Abstract:** In an aplanatic optical system, where the entrance pupil is not located at the front focal plane, a corresponding quadratic phase is expected at the back focal plane. However, tightly focused optical fields evaluated by methods based on the traditional Richards-Wolf approach, lacks this quadratic phase. In the current work we calculate the focused field, for both high and medium numerical apertures, based on the Stratton-Chu diffraction integral in a 2D system. We find that the quadratic phase factor depends on the numerical aperture, and it approaches the corresponding paraxial value for lower numerical apertures.

## References and links

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## 1. Introduction

Tight focusing of light in high numerical aperture (NA) systems is becoming increasingly important with recent advances in nanotechnology. The majority of methods suggested for the tight focusing of different types of illumination are based on the approach pioneered by Richards and Wolf in 1959 [1] (for a detailed review of these methods see [2]). Although this approach has a number of drawbacks it is the only one that offers reasonable 3D computation complexity for this class of diffraction problems. Among its drawbacks is the fact that the derived focused field at the back focal plane lacks the expected quadratic phase when the entrance pupil is not situated at the front focal plane.

The computational complexity can be substantially reduced when 2D diffraction problems are considered. Therefore these problems lend themselves to treatment by other rigorous diffraction approaches, which involve less assumptions and approximations than the Richards-Wolf (RW) approach. One such approach is the calculation of the diffracted field by means of the Stratton-Chu (SC) [3] diffraction integral.